

1. Problem Statement

Environment

- Bounded rectangle
 - Tessellated to the square grid
 - Grid cell size is $2r$

Agents

- Open disks of radii r
- Agents positions = centers of the cells

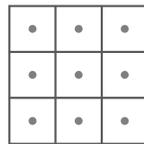
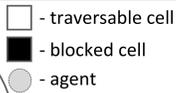
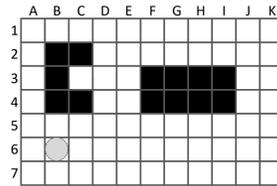
Actions

- Move
 - Any-angle moves allowed
 - Moving speed is constant and equals $2r$ per time step
 - Time is equivalent to distance traveled
 - Inertial effects are neglected
- Wait (before the move)
 - An arbitrary amount of time
 - Occupying the cell

Task

Given start and goal locations find a set of feasible conflict-free trajectories

Solution cost = sum of individual costs



Challenges raised by any-angle moves

- Conflict is not associated with the exact grid cell or edge
 - Conflict may occur at any moment of contiguous timeline
-

2. Suggested Approach

Decoupled, e.g. prioritized planning

- Agents are assigned unique priorities
 - Trajectories are planned one-by-one
- Previously planned trajectories = moving obstacles

Individual planner (AA-SIPP)

- Builds on top of SIPP algorithm [1]
- Handles any-angle moves
- Incorporates "avoid low-priority start locations" strategy [2]
- In order to guarantee the completeness of the multi-agent planner in specific cases

Resultant multi-agent planner AA-SIPP(m)

- Is complete under well-defined conditions, which often hold in practice
- Is not complete in general

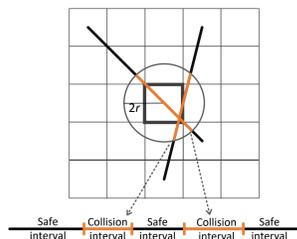
3. SIPP/AA-SIPP Search Space

Search node identifier $s=[cfg, interval]$

- cfg – configuration (grid cell)
- $interval$ – the contiguous period of time, during which an agent may safely occupy cfg

Search node associated data $g(s), h(s), parent(s), time(s)$

- $time$ – earliest possible arrival time to cfg via $parent$ (must be within $interval$)
- g – cost of the best path to s found so far
- h – heuristic estimate of cost-to-go



4. Any-Angle SIPP (AA-SIPP)

Algorithm: AA-SIPP

```

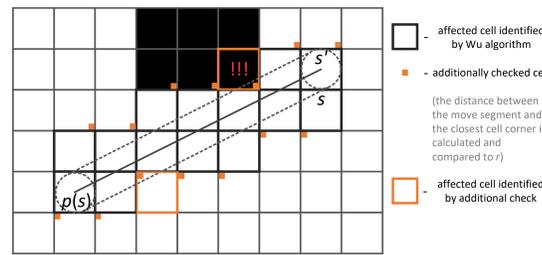
1   $g(s_{start}) = 0; OPEN = \emptyset;$ 
2  insert  $s_{start}$  into  $OPEN$  with  $f(s_{start}) = h(s_{start});$ 
3  while  $s_{goal}$  is not expanded do
4  remove  $s$  with the smallest  $f$ -value from  $OPEN;$ 
5  for each  $cfg$  in  $NEIGHBORS(s.cfg)$  do
6   $successors = getSuccessors(cfg, s);$ 
7  if  $cfg$  is reachable from  $parent(s).cfg$  then
8   $successors = successors \cup getSuccessors(cfg, parent(s));$ 
9  for each  $s'$  in  $successors$  do
10 if  $s'$  was not visited before then
11  $f(s') = g(s') = \infty;$ 
12 if  $g(s') > g(s) + c(s, s')$  then
13  $g(s') = g(s) + c(s, s');$ 
14 update  $Time(s');$ 
15 insert  $s'$  into  $OPEN$  with  $f(s') = g(s') + h(s');$ 
16 Function  $getSuccessors(cfg, s)$ 
17  $successors = \emptyset;$ 
18  $m.time = time$  to reach  $cfg$  from  $s.cfg;$ 
19  $start.t = time(s) + m.time;$ 
20  $end.t = endTime(interval(s)) + m.time;$ 
21  $intervals = get$  all safe intervals for  $cfg;$ 
22 for each safe interval  $i$  in  $intervals$  do
23 if  $startTime(i) > end.t$  or  $endTime(i) < start.t$  then
24 continue;
25  $t = earliest$  arrival time from  $s$  to  $cfg$  during interval  $i$  with no collisions;
26 if  $t$  does not exist then
27 continue;
28  $s' = state$  of configuration  $cfg$  with interval  $i$  and time  $t;$ 
29 insert  $s'$  into  $successors;$ 
30 return  $successors;$ 
    
```

AA-SIPP

- Modified SIPP algorithm
 - Uses shortcuts (like Theta* [3])
- Is complete w.r.t. grid discretization of the workspace
- Finds solutions which cost does not exceed the cost of regular SIPP solutions

Validating the shortcut

- Identify all cells affected by the moving agent
 - Wu algorithm [4] plus
 - Additional checks
- Check whether the identified cells are all unblocked
 - If they are – the shortcut move is valid
 - If they are not – the shortcut move is invalid



5. Estimating Earliest Arrival Times

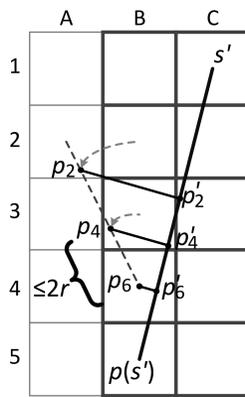
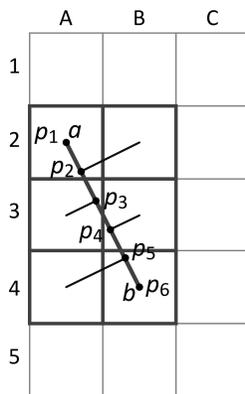
Representation of moving obstacles

- Moving obstacles' trajectories are split into the sequences of constraints: $constraint = [p, time(p)],$
 - p – point on the trajectory
 - $time(p)$ – moment of time when the obstacle passes p
- Constraints are tied to the grid cells affected by moving obstacle
- Several constraints are stored if moving obstacle waits at the cell
- Distance (and time difference) between each two consecutive constraints does not exceed $2r$

Algorithm: Calculation of EAT

```

1   $CELLS = get$  all cells affected by  $\langle p(s'), s' \rangle;$ 
2   $INTERVALS = \emptyset;$ 
3  for each  $cell$  in  $CELLS$  do
4  for each  $constraint \in cell$  do
5  if  $distance(constraint, \langle p(s'), s' \rangle) < 2r$  then
6   $offset = distance(s',$  the closest to  $constraint$  point on  $\langle p(s'), s' \rangle;$ 
7   $collision\_interval = [time(constraint) - 4r + offset, time(constraint) + 4r + offset];$ 
8   $INTERVALS = INTERVALS \cup collision\_interval;$ 
9  else
10 continue;
11  $t = time(p(s')) + distance(p(s'), s');$ 
12  $INTERVALS = combine$  all intersecting collision intervals in  $INTERVALS;$ 
13 for each  $collision\_interval$  in  $INTERVALS$  do
14 if  $t \in collision\_interval$  then
15  $t = endTime(collision\_interval);$ 
16 if  $t \notin safe$  interval of  $s'$  then
17  $s'$  is not reachable from  $p(s')$  during the consistent safe interval;
18 else
19  $t$  is the earliest arrival time;
    
```



6. AA-SIPP(m)

Algorithm: AA-SIPP(m)

```

1   $PATHS = \emptyset;$ 
2  determine agents' priorities;
3  for each agent w.r.t priorities do
4   $path = find$  path for agent with AA-SIPP;
5  if  $path \neq \emptyset$  then
6   $PATHS = PATHS \cup path;$ 
7  transform  $path$  into the sequence of constraints and add them to the search space;
8  return  $PATHS;$ 
    
```

AA-SIPP(m) is complete in well-formed infrastructures (WFIs)

WFI is a planning instance having the following property:

- Individual path for each agent exists:
 - with at least r -clearance with respect to the static obstacles
 - with at least $2r$ -clearance to any other start or goal location

7. Experimental Evaluation

Algorithms

- ICBS [5]
- SIPP(m)
- AA-SIPP(m)
- ECBS [6]

The 1st experiment

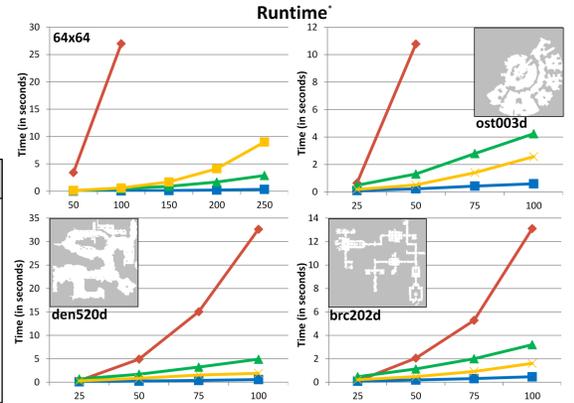
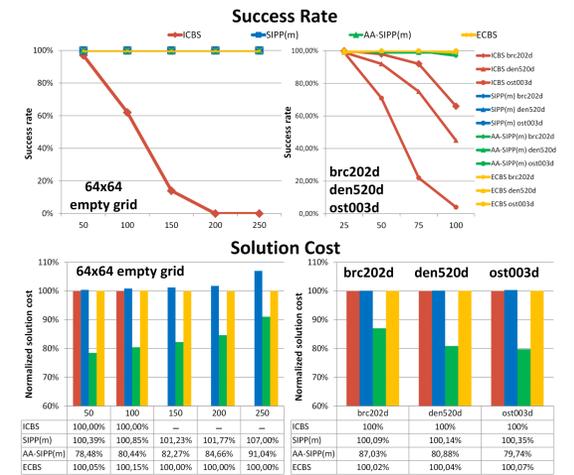
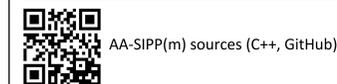
- Empty 64x64 grid
- Number of agents: 50, 100, 150, 200 and 250
- 100 instances per each number of agents (500 in total)
- Each instance is a well-formed infrastructure
- Runtime limit – 5 minutes

The 2nd experiment

- 3 grid maps from MovingAI benchmark set (Dragon Age: Origins):
 - brc202d
 - den520d
 - ost003d
- Number of agents: 25, 50, 75 and 100
- 100 instances per map per number of agents (1200 in total)
- Runtime limit – 5 minutes

Results

- AA-SIPP(m) solutions are up to 22% better (solution costs are lower)
- AA-SIPP(m) scales well to large number of agents
- AA-SIPP(m) success rate exceeds 97%



* runtimes are not directly comparable due to different implementations of SIPP(m), AA-SIPP(m) and ICBS, ECBS.

Key References

- Phillips, M.; and Likhachev, M. 2011. SIPP: Safe interval path planning for dynamic environments. In Proceedings of The 2011 IEEE International Conference on Robotics and Automation (ICRA-2011), 5628–5635.
- Čáp, M.; Novák, P.; Kleiner, A.; and Selecký, M. 2015. Prioritized planning algorithms for trajectory coordination of multiple mobile robots. IEEE Transactions on Automation Science and Engineering 12(3):835–849.
- Nash, A.; Daniel, K.; Koenig, S.; and Felner, A. 2007. Theta*: Any-angle path planning on grids. In Proceedings of The 22nd AAAI Conference on Artificial Intelligence (AAAI-2007), 1177–1183.
- Wu, X. 1991. An efficient antialiasing technique. In Proceedings of The 18th Annual Conference on Computer Graphics and Interactive Techniques (SIGGRAPH-1991), 143–152.
- Boyarski, E.; Felner, A.; Stern, R.; Sharon, G.; Betzalel, O.; Tolpin, D.; and Shimony, E. 2015. ICBS: Improved conflict-based search algorithm for multiagent pathfinding. In Proceedings of The 24th International Joint Conference on Artificial Intelligence (IJCAI-2015), 740–746.
- Barer, M.; Sharon, G.; Stern, R.; and Felner, A. 2014. Suboptimal variants of the conflict-based search algorithm for the multi-agent pathfinding problem. In Proceedings of The 7th Annual Symposium on Combinatorial Search (SoCS-2014), 19–27.